Intervention in \textit{tough}-constructions

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1 \textbf{Introduction}

- \textit{A tough history}
  Ever since the early days of generative syntax, \textit{tough}-predicates have proven considerably elusive to analysis (e.g. Lees 1960; Chomsky 1964). \textit{Tough}-predicates occur in one of two structural frames which are nearly synonymous in meaning:

1. \textit{Tough-}construction
   The matrix subject is obligatorily interpreted as coreferential with the non-subject gap position in the embedded nonfinite clause:

   (1) Alex, is tough to please \textit{___}.i.

2. Expletive construction
   The matrix-subject position is filled by the expletive it:

   (2) It is tough to please Alex.

- \textit{Embedded $\overline{\Lambda}$-movement}
  It is generally taken for granted that there is $\overline{\Lambda}$-movement within the lower clause. First, for theory-internal reasons, the $\overline{\Lambda}$-movement accounts for why the embedded subject does not intervene for relativised minimality. Second, \textit{tough}-constructions pass diagnostics for $\overline{\Lambda}$-movement (Chomsky 1977, 1982):

   (3) \textbf{a. Embedded clause forms a wh-island}
       \begin{itemize}
       \item [\textit{What sonatas $j$ is this violin easy \textit{[Op$_i$ to play $t_j$ on $t_i$]}}?]
       \end{itemize}

   \textbf{b. Cannot cross a wh-island}
       \begin{itemize}
       \item [\textit{John is difficult \textit{[Op$_i$ to imagine Mary \textit{[wondering whether she would marry $t_j$]}}}]]
       \end{itemize}

   \textbf{c. Subject to Complex-NP Constraint}
       \begin{itemize}
       \item [\textit{John is easy \textit{[Op$_i$ to describe to Bill \textit{[a plan to assassinate $t_j$]}}}]]
       \end{itemize}

   \textbf{d. Licenses parasitic gaps
       \begin{itemize}
       \item [\textit{Which files $i$, are easy \textit{[Op$_i$ to discard $t_j$ \textit{[without reading pg]}}}?]
       \end{itemize}

- \textit{The matrix subject}
  Analyses differ in how this embedded movement is linked to the matrix subject position.

1.1 \textbf{Two accounts of \textit{tough}-constructions}

- \textit{How this embedded movement dependency is linked to the surface subject has been a matter of controversy. Analyses of \textit{tough}-constructions divide into two groups: the long-movement analysis and the base-generation analysis.}

1. \textit{Long-movement analysis}
   The matrix subject originates in the embedded gap position. First, it $\overline{\Lambda}$-moves to the edge of the embedded clause. Second, it subsequently $\Lambda$-moves to the matrix subject position.

   (4) \begin{itemize}
   \item [\textit{Alex$_i$ is tough \textit{[t$_i$ PRO$_{ARB}$ to please $t_i$]}}]
   \end{itemize}

   \[ \Rightarrow \text{E.g. Rosenbaum (1967); Postal (1971); Postal \& Ross (1971); Brody (1993); Hornstein (2001); Hicks (2009); Hartman (2011, 2012a,b)} \]

2. \textit{Base-generation analysis}
   The matrix subject is base-generated in that position. A null operator $\overline{\Lambda}$-moves from the gap position to the edge of the embedded clause. The matrix subject is interpreted as coreferential with the null operator.

   (5) \begin{itemize}
   \item [\textit{Alex$_i$ is tough \textit{[Op$_i$ PRO$_{ARB}$ to please $t_i$]}}]
   \end{itemize}
1.2 Defective intervention in tough-constructions

- **Hartman’s new argument for long movement**
  Hartman (2011, 2012a,b) observes that an experiencer PP occurring between the tough-predicate and the infinitival clause yields ungrammaticality in the tough-construction, but not in the expletive construction.

\[
\begin{align*}
\text{(6) } & \text{a. It is important } [\text{PP to Mary}] \text{ to avoid cholesterol.} \\
& \text{b. *Cholesterol, is important } [\text{PP to Mary}] \text{ to avoid } \_\_\_.
\end{align*}
\]

Hartman proposes that this is an instance of **defective intervention**, whereby an element with inactive syntactic features blocks agreement across it with a lower active element (Chomsky 2000).

⇒ He argues that this supports the long-movement analysis because defective intervention is a purported property of A-movement (Chomsky 2000). The PP is a defective intervenor for the final A-movement step:

\[
\begin{align*}
\text{(7) Cholesterol is important } [\text{PP to Mary}] [\pi \text{ PRO}_{\text{ARR}} \text{ to avoid } t_i] \\
\end{align*}
\]

1.3 Overview

- **Claim made in this talk**
  We argue that upon closer scrutiny the PP intervention facts do not support a long-movement account. We propose a base-generation analysis of tough-constructions and that the intervention of the experiencer PP follows from the compositional semantics of tough-predicates.

- **Structure of this talk**
  1. We review Hartman’s Generalisation that the matrix adjective of a tough-construction cannot occur with an experiencer PP.
  2. We show that the empirical landscape is broader: Adjuncts in general cannot intervene between a tough-predicate and the embedded clause in tough-constructions.
  3. We propose a semantics for tough-predicates, which accounts for these intervention facts.
  4. Our semantic account extends without stipulation to many other known properties of tough-predicate, many of which have remained elusive under long movement accounts.

2 Hartman’s Generalisation

- **A tale of two for-phrases**
  It is well-known since at least Chomsky (1973) that two for-phrases can occur in the expletive construction, but that only one can occur in the corresponding tough-construction:

\[
\begin{align*}
\text{(8) a. It is easy for the rich for the poor to do the work.} \\
& \text{b. The work, is easy for the rich (*for the poor) to do } \_\_\_.
\end{align*}
\]

- **Interpretation of the PPs**
  - In (8a), the first for-phrase for the rich modifies the adjective easy and the second for-phrase for the poor is an embedded infinitival subject.
  - All else equal, the for-phrase that remains in the tough-construction (8b) is interpreted as both the modifier and the embedded subject.

⇒ This means that the meaning of (8b) does not make it clear where the for-phrase sits.

- **The traditional view**
  Before Hartman (2011, 2012a,b), it was standardly assumed that the for-phrase to survive in a tough-construction modifies the adjective and is not an embedded infinitival subject (e.g. Faraci 1974; Lasnik & Fiengo 1974; also see Rezac 2006 and Ahn & Sailor 2014 for recent proposals to this effect).

\[
\begin{align*}
\text{(9) a. The work, is easy } [\text{PRO}_{\text{ARR}} \text{ to do } \_\_\_.] \\
& \text{b. *The work, is easy } [\text{for the rich to do } \_\_\_.]
\end{align*}
\]

- **Hartman’s contribution**
  Hartman shows that this traditional assumption is untenable. We review his evidence below.
1. **Unambiguous PPs**
   When the experiencer phrase is headed by a preposition other than *for*, this experiencer phrase cannot occur in a *tough*-construction. Crucially, these prepositions cannot introduce an embedded individual subject.

   (10) a. It is important (*to Mary*) to avoid cholesterol.
   b. Cholesterol, is important (*to Mary*) to avoid ____.

   (11) a. It was very hard (*on me*) to give up sugar.
   b. Sugar, was very hard (*on me*) to give up ____.

2. **Scope**
   A *for*-phrase can take scope above or below the matrix adjective in the expletive construction (12a). This corresponds to the two construals: (i) embedded subject or (ii) adjectival modifier. In the corresponding *tough*-construction, only low scope is possible (12b). This indicates that the *for*-phrase must be parsed as the embedded subject.

   (12) a. It is impossible *for every student* to fail this class.
      
      (impossible >> every student; every student >> impossible)
   b. This test, is impossible *for every student* to fail ____.
      (impossible >> every student; *every student >> impossible)

3. **Crosslinguistic evidence**
   In languages where infinitival subjects cannot be introduced by a preposition, a PP must modify the adjective. In a *tough*-construction, such a PP is disallowed:

   (13) **Italian**
   a. È impossibile (*per gli studenti*) capire questi problemi.
      is impossible for the students understand these problems
      'It is impossible (for the students) to understand these problems.'
   b. Questi problemi, sono impossibile (*per gli studenti*) da
capire ____.
      these problems are impossible for the students DA
derstand ____.
      'These problems are impossible (*for the students) to understand.'
      [Hartman 2012a:123]

   • There is no general incompatibility between *tough*-constructions and experiencers:

     (14) a. *[PP For Mary ] the book, was fun to read ____.
     b. The book, was fun to read ____ *[PP for Mary ].

   * **Generalisation**
   Hartman concludes from these facts that the following generalisation holds:

     (15) **Hartman’s Generalisation**
     In a *tough*-construction, no experiencer phrase can intervene between the *tough*-predicate and the infinitival clause.

   • **Hartman’s analysis: Defective intervention**
   Hartman takes Hartman’s Generalisation (15) as evidence for A-movement of the *tough*-subject from inside the infinitival clause to the matrix subject position. By hypothesis, an experiencer PP serves as a defective intervenor for A-movement across it.

     (16) Cholesterol is important *[PP to Mary ] [ t_i PRO}_{\text{ARR}} to avoid t_i ]

   • **Intervention in A-raising**
   This groups the intervention effects in *tough*-constructions with purported intervention effects in Romance A-raising:

     (17) **Italian**
     a. Sembra (*a Maria) che Gianni è stanco.
        
        seems to Maria that Gianni is tired
        'It seems to Maria that Gianni is tired.'
     b. Gianni, sembra (*a Maria) essere stanco ____.
        Gianni seems to Maria to be tired ____
        'Gianni seems to Maria to be tired.'
        [Hartman 2012a:122]

   * **Summary**
   In a *tough*-construction, the *tough*-predicate cannot be modified by an experiencer PP. Hartman (2011, 2012a,b) argues that this constitutes an argument for a long-movement analysis: The PP acts as a defective intervener and blocks A-movement over it.
3 Problems for a defective intervention account

- **Section outline**
  In this section, we show that the empirical landscape is more complicated.
  1. PPs do not intervene for A-movement in English.
  2. The same intervention effects observed by Hartman occur in structures not involving movement.
  3. The intervention effects generalise to adjuncts which do not trigger intervention effects in known cases of A-movement.
  4. There is an argument/adjunct distinction that remains mysterious under a defective intervention account.

3.1 PPs are not A-interveners

- As Hartman (2011) himself observes, PPs do not intervene for A-raising in English:

  \[(18) \text{John seems } [\text{PP to Mary}] \quad t \quad \text{to be happy.} \]

  \[\text{A}\]

  This observation undermines the basic claim that PP intervention diagnoses A-movement in English.

3.2 PP intervention in nonmovement structures

- PP intervention also emerges in constructions lacking a corresponding expletive version from which they could be derived via movement.

  **Pretty-predicates**
  Adjectives like *pretty* or *tasty* can occur in a *tough*-construction, but not the expletive construction:

  \[(19) \text{a. Marigolds} \quad \text{are } \text{pretty } \text{to look at }\_\_\_. \]

  \[\text{b. } \text{*It is } \text{pretty } \text{to look at marigolds.} \]

  \[\text{c. Oatmeal} \quad \text{is } \text{tasty } \text{to eat }\_\_\_. \]

  \[\text{d. } \text{*It is } \text{tasty } \text{to eat oatmeal.} \]

- However, although a *pretty*-predicate can in principle occur with an experiencer phrase, the experiencer phrase cannot occur between the predicate itself and the embedded clause:

  \[(20) \text{a. Mary is pretty } [\text{PP to John}]. \]

  \[\text{b. } \text{*Mary} \text{is pretty } [\text{PP to John} \text{ to look at }\_\_\_.] \]

  \[\text{c. } [\text{PP To John}], \text{Mary is pretty to look at }\_\_\_.] \]

- **Gapped degree phrases**
  Like *pretty*-predicates, gapped degree phrases (GDPS) can occur in a *tough*-construction, but lack the expletive counterpart:

  \[(21) \text{a. This table} \quad \text{is } \text{too heavy to lift }\_\_\_. \]

  \[\text{b. } \text{*It is } \text{too heavy to lift this table.} \]

- GDPS can be modified by an experiencer phrase, but a *for*-phrase between the adjective and the infinitival clause is in principle ambiguous. However, while (22b) shows that a *for*-phrase is able to modify the matrix predicate, scope reveals that it cannot do so when it is between the predicate and the embedded clause (22c).

  \[(22) \text{a. The table} \quad \text{is too heavy to lift }\_\_\_[PP \text{ for only one worker}]. \]

  \[\text{(only one } \gg \text{ too heavy; too heavy } \gg \text{ only one)} \]

  \[\text{b. } [\text{PP For only one worker}] \text{ the table} \quad \text{is too heavy to lift }\_\_\_. \]

  \[\text{(only one } \gg \text{ too heavy; }^{*}\text{too heavy} \gg \text{ only one)} \]

  \[\text{c. The table} \quad \text{is too heavy } [\text{PP for only one worker}] \text{ to lift }\_\_\_. \]

  \[^{*}(\text{only one } \gg \text{ too heavy; too heavy } \gg \text{ only one)} \]

  \[\Rightarrow \text{ In GDP, an intervening *for*-phrase must be within the embedded clause. It cannot modify the matrix predicate.} \]

- **Corroborating evidence from Italian**
  A PP that unambiguously attaches to the matrix predicate is unavailable in gapped degree phrases:
3.3 Intervention of noninterveners

- Bruening (2014) observes that it is not only experiencer phrases that are illicit in tough-construction, but adjuncts in general:

  (24) a. It is always annoying [PP at meetings] to talk about the budget.
  
  b. *The budget, is always annoying [PP at meetings] to talk about ____i.

- Crucially, these PPs do not otherwise block A-movement:

  (25) John seemed [PP at the meeting] to be agitated.

- Italian

  The same is true for Italian:

  (26) a. È difficile [PP al crepuscolo] vedere questi colori
  
  is difficult at.the twilight to see these colors
  
  'It is difficult at twilight to see these colors'

  b. *Questi colori, sono difficili [PP al crepuscolo] da vedere
  
  these colors are difficult at.the twilight DA to see
  
  ____i

  'These colors are difficult at twilight to see' [Bruening 2014:71f.]

- Extension to pretty-predicates and GDP

The same intervention effects of adjuncts in general can be observed for pretty-predicates and GDPs:

  (27) a. Mary will be pretty [PP at her wedding] to look at ____i.
  
  b. *Mary, will be pretty [PP at her wedding] to look at ____i.

  (28) a. [PP In this awkward position] the table, will be too large to lift ____i.
  
  b. *The table, will be too large [PP in this awkward position] to lift ____i.

* Conclusion

That the intervention effects generalise to adjuncts, not just experiencer PPs, discredits assimilating these effects to defective intervention. These elements intervene in tough-constructions despite demonstrably not intervening for A-movement. Consequently, the intervention cannot be the result of A-movement over them.

3.4 Non-intervening PPs

- Finally, only adjunct PPs act as intervener. Argument PPs do not, as in (29):

  (29) a. John, is too fond [PP of Mary] to like ____i.
  
  b. John, is too angry [PP at Mary] to invite ____i.

  ⇒ On a defective intervention account, no such asymmetry is expected.

3.5 Section summary

- We have presented new evidence that shows that Hartman’s Generalization should be revised and argued that a more comprehensive assessment of the restriction is problematic for Hartman (2011)’s defective intervention account:

  1. PPs do not cause intervention in structures that unambiguously involve A-movement in English.

  2. PP intervention also arises in structures that lack an expletive counterpart and are thus not derived by A-movement.

  3. Adjunct PPs that demonstrably do not intervene for A-movement nonetheless give rise to intervention in tough-constructions.

  4. PPs do not intervene if they are an argument of the tough-predicate.
• We propose the revised generalisation in (30):

(30) **Revised Hartman’s Generalisation**
In tough-constructions, gapped degree phrases and pretty-predicates, no adjunct may occur between the adjective and the infinitival clause.

* Conclusion
It is unclear how this extended generalisation could be captured on a defective intervention account. The next section shows how the facts follow from a base-generation account.

4 Proposal

• **Analysis in a nutshell**
  - Tough-predicates come in two variants: (i) one version that combines with a proposition, corresponding to the expletive construction, and (ii) another version that combines with a property of individuals, corresponding to the tough-construction.
  - In a tough-construction, the embedded clause is a null-operator structure wherein a null operator Π-moves to the clause edge triggering abstraction over an individual variable, thus forming a property of individuals:

(31) $\text{XP is tough} \begin{array}{c}
\Pi \\
\text{tough[Op}_{t_i} \text{[PRO}_{a_{\text{ex}}\text{ to please } t_i} ]}
\end{array}$
$\text{\Rightarrow LF: XP is tough } \lambda x[\text{PRO}_{a_{\text{ex}}\text{ to please } x}]

  - Intervention effects result from an irresolvable semantic-type mismatch. The property-taking version of a tough-predicate yields an AP that itself denotes a property of individuals. Therefore, it cannot combine with experiencer PPs and adjuncts, which only combine with propositions.

• To simplify the discussion, the focus will be on tough-predicates, but everything that follows applies equally to pretty-predicates and GDPs. Pretty-predicates only have a property-taking version. GDPs are discussed in the appendix.

4.1 Semantics of tough-predicates

• **A judge-based semantics**
Tough-predicates have the property that their truth is evaluated with respect to some individual.

• To capture this property, we propose a judge-based semantics of tough-predicates wherein they are subjective to the JUDGE of the proposition. This is represented as an argument $j$ of the denotation function (following Stephenson 2007, 2010; Lasersohn 2005).

• **Two versions of tough**
Tough-predicates come in two variants: (i) a proposition-taking version (32), which corresponds to the expletive construction, and (ii) a property-taking version (33), which corresponds to the tough-construction:

(32) **Expletive-construction variant** $(s, (d, st))$

$\text{[tough}_{\text{expl}]^j} =$
$\lambda p, t \lambda d \lambda w_s \cdot s \in ACC_{w, j} \text{TOUGH}_{w, j'}\left([p]^{j'}(w')\right)(d)$
(In all of $j$’s doxastic alternatives, $p$ is $d$-tough to $j$.)

(33) **Tough-construction variant** $(\langle e, st \rangle, \langle (d, e, st) \rangle)$

$\text{[tough}_{\text{tc}]^j} =$
$\lambda Q_{(e, st)} \lambda d \lambda x \lambda w_s . \forall (w', j') \in ACC_{w, j} \text{TOUGH}_{w, j'}\left(\llbracket Q \rrbracket^{j'}(x)(w')\right)(d)$
(In all of $j$’s doxastic alternatives, $Q(x)$ is $d$-tough to $j$.)

(34) $ACC_{w, x}$
$\{\langle w', y \rangle : \text{it is compatible with what } x \text{ believes in } w \text{ for } x \text{ to be } y \text{ in } w'\}$

(35) $\text{TOUGH}_{w, j}(p)(d) \iff p \text{ is at least } d \text{-tough to } j \text{ in } w$

• Both tough$_{\text{expl}}$ and tough$_{\text{tc}}$ assert that the proposition $p$ (saturated) or $Q(x)$ (to be saturated) is $d$-tough according to the judge $j$ in all the centred worlds (world–individual pairs) where the judge $j$ is the centre.

---

1 The semantics that we propose treat tough-predicates as predicates of personal taste, adopting proposals from Stephenson (2007, 2010) about predicates of personal taste and control. See the appendix for more details.

2 We assume that tough$_{\text{tc}}$ combines with the degree variable before the tough-subject; this simplifies the semantics somewhat. However, nothing critical hinges on this decision.
The crucial distinction

tough_{expl} and tough_{rc} differ in the arguments that they combine with. Setting aside the degree and world arguments:

- tough_{expl} combines with a single propositional argument \( \lambda p \).
- tough_{rc} combines first with an argument denoting a property of individuals \( \lambda Q \) and then with an individual argument \( \lambda x \). The predicate Q is subsequently saturated by \( x \).

• PRO and the judge

The embedded clause of a tough-predicate contains PRO, which is obligatorily coreferential with the judge of the tough-predicate (Bhatt & Izvorski 1997; Epstein 1984; Lebeaux 1984; Bhatt & Pancheva 2006):

\[(36) \quad \text{Implicit judge (with generic interpretation) } \rightarrow \text{PRO}_{\text{arb}}
\]

It is fun [ \( \text{PRO}_{\text{arb}} \) to play hockey]

\(~ \rightarrow \text{gen } x \) it is fun for \( x \) for \( x \) to play hockey

\[(37) \quad \text{Overt judge, } \rightarrow \text{PRO}_i (37a) \neq (37b)
\]

a. It was tough [ on Mary, ] [ \( \text{PRO}_{i,j} \) to avoid cholesterol]

\(~ \rightarrow\) It was tough on Mary for Mary to avoid cholesterol

b. It was tough [ on Mary ] [ for Bill to avoid cholesterol]

• For example, (37a) cannot have an interpretation analogous to (37b) in which PRO would refer to Bill. Rather, PRO must refer to Mary.

• This generalisation is captured by PRO referring directly to the judge \( j \), as independently proposed by Stephenson (2007, 2010).\(^5\)

\[(38) \quad [\text{PRO}]^j = j \quad \text{[Stephenson 2007, 2010]}
\]

4.2 Illustrating the semantics

• Expletive construction

In the expletive construction, the tough-predicate first takes a propositional argument \( \lambda p \) and then a degree argument \( \lambda d \):

\[(39) \quad \text{It is fun [ \( \text{PRO} \) to play hockey]}
\]

\[\begin{array}{c}
\exists \\
\text{AP}_3 : st \\
\text{AP}_2 : st \\
d \quad \text{AP}_1 : (d, st) \\
\text{fun}_{\text{expl}} \quad \text{CP : st} \\
\text{PRO to play hockey}
\end{array}\]

a. \([\text{CP}]^j = \lambda w . \text{PLAY}(\text{hockey})(j)(w)
\]

(Proposition: \( j \) plays hockey.)

b. \([\text{fun}_{\text{expl}}]^j = \lambda p_{st} \lambda d_{st} \lambda w_{st} . \forall \langle w', f' \rangle \in \text{ACC}_{w,f} \left[ \text{FUN}_{w', f'}(\langle p \rangle^j(\langle w' \rangle))(d) \right] \\
\]

(\( j \) finds \( p \) to be \( d \)-fun.)

c. \([\text{AP}_3]^j = \lambda w . \exists d \forall \langle w', f' \rangle \in \text{ACC}_{w,f} \left[ \text{FUN}_{w', f'}(\langle \text{PLAY}(\text{hockey})(f')(w') \rangle)(d) \right] \\
\]

(Proposition: \( j \) finds \( j \) playing hockey to be \( d \)-fun.)

• In (39), the derivation proceeds as follows:

1. The tough-predicate fun combines with the CP, which denotes a proposition.
2. Second, it combines with the degree variable \( d \).
3. Third, existential closure applies over the degree variable (Heim 1985).\(^4\)

• The judge parameter \( j \) is then interpreted either generically or as referring to some contextually salient individual(s).

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\(^5\) Some of Stephenson’s motivations for this proposal are given in the appendix.

\(^4\) Existential quantification over the degree variable yields too weak of truth conditions for a positive degree construction. We assume that an operator like Rett’s (2008) \textsc{eval} anchors the degree variable to a contextually valued standard.
• **Tough-construction**

In the tough-construction, the tough-predicate first combines with an argument denoting a property of individuals \(\lambda Q\), then a degree argument \(\lambda d\), and then an individual argument \(\lambda x\).

- The embedded clause is a null-operator structure formed by \(\overline{A}\)-movement of the null operator from the gap position to the clause edge. The null operator is interpreted as a \(\lambda\)-abstraction over its trace (Nissenbaum 2000):

\[
(40) \quad [\text{Op}_1[...t_1...]] \leadsto \text{LF}: \lambda x[...x...]
\]

Therefore, the null-operator structure denotes a property of individuals and is of the appropriate semantic type to compose with the tough-construction variant of a tough-predicate:

\[
\begin{align*}
\text{(41) } \text{Hockey is fun} & \left[ \text{Op}_1 \text{ PRO to play } \_\_ \_ \_ \_ \_ \right] \\
\end{align*}
\]

- In (41), the derivation proceeds as follows:
  1. The tough-predicate \(\text{fun}\) combines with the CP, which denotes a property of individuals.
  2. Second, it combines with the degree variable \(d\).
  3. Third, it combines with \(\text{hockey}\) and ultimately saturates the CP with it.
  4. Last, existential closure applies over the degree variable.\(^5\)

### 4.3 Intervention is a semantic-type mismatch

- **Semantics of the experiencer PP**

  The role of the experiencer PP is to overtly specify the judge. Let us assume that it is introduced by the functional head \(\text{Appi}^0\):

\[
(42) \quad \left[\text{Appi}^0\right] = \lambda p \lambda j'' \lambda w \cdot \left[ p \right]''(w) \quad \{st, \langle e, st \rangle\}
\]

(Shift the judge of the proposition \(p\) to \(j''\).)

- In (42), \(\text{Appi}^0\) combines first with a propositional argument \(\lambda p\) and then with the experiencer PP \(\lambda j''\). It shifts the judge argument of the denotation function for \(p\) to the explicit individual(s) denoted by the experiencer PP.

\(\Rightarrow \) Only attaches to propositions

  Crucially, \(\text{Appi}^0\) only combines with propositions. This can be observed outside the context of tough-predicates with predicates of personal taste, which are also judge-dependent:

\[
(43) \quad a. \quad \left[ [\text{PP To Kyle,}] \right] \left[ \text{the rutabagas are tasty} \right]
\]

- b. \([ [\text{The rutabagas are tasty} ] [\text{PP to Kyle} ] ]\)

- c. * The rutabagas are \([\text{PP to Kyle} ]\) tasty.

- In (43a) and (43b), the experiencer PP attaches high in the structure, at the propositional level, and is linearised to either the left or the right. (43c) shows that the experiencer PP cannot attach in a medial, nonpropositional position.

---

\(^5\) As existential closure is generally available only at the level of the proposition, the degree variable is a free variable until the individual argument of \(\text{fun}_{\text{rc}}\) is saturated and existential closure can apply (e.g. Heim 1982; Partee 1987).
**Extended adjectival projection**
The structure of the extended adjectival projection is in (44). \(\text{App}^0\) selects for an AP and is itself selected by \(aP\). In this sense, the extended adjectival projection parallels the extended verbal projection.

\[
(44) \quad [aP \ldots a^0 [\text{App} \ldots \text{App}^0 [A^0 \ldots]]]
\]

We make the following two assumptions:
- The correct linear order of the experiencer PP with respect to the adjective is derived by head movement of \(A^0\) to \(a^0\).  
- The tough-subject is merged in [Spec, \(aP\)], parallel to the external argument merging in [Spec, vP].

 ✓ Expletive construction
In the expletive construction, as the AP denotes a proposition, it can successfully combine with \(\text{App}^0\) and therefore with an experiencer PP:

\[
(45) \quad \text{It is fun } [\text{for Lucy}] [\text{PRO to play hockey}]
\]

\[
\exists aP_2 : st \quad \exists aP_1 : st \quad \text{App}^0 : st \quad \text{PP} : e \quad \text{App}^0 : (e, st) \quad \text{AP}_2 : st \\
\text{for Lucy} \quad \text{App}^0 : (e, st) \quad \text{AP}_1 : (d, st) \\
\text{fun}_{\text{expl}} \quad \text{CP} : st \quad \text{PRO to play hockey}
\]

\[\begin{align*}
\text{a. } [\text{AP}_2]^j &= \lambda w. \forall (w', j') \in \text{ACC}_{w,j} \left[ \text{Fun}_{w,j'} \left( \text{play(hockey)}(j')(w')(d) \right) \right] \\
&= \text{(Proposition: Lucy finds Lucy playing hockey to be}\ d\text{-fun.)}
\end{align*}\]

\[\begin{align*}
\text{b. } [\text{App}^0]^j &= \lambda j' \lambda w. \forall (w', j') \in \text{ACC}_{w,j'} \left[ \text{Fun}_{w,j'} \left( \text{play(hockey)}(j')(w')(d) \right) \right] \\
&= \text{(The set of x such that x finds x playing hockey to be } d\text{-fun.)}
\end{align*}\]

\[\begin{align*}
\text{c. } [aP_2]^j &= \lambda w. \exists d \forall (w', j') \in \text{ACC}_{w,Lucy} \left[ \text{Fun}_{w', j'} \left( \text{play(hockey)}(j')(w')(d) \right) \right] \\
&= \text{(Proposition: Lucy finds Lucy playing hockey to be } d\text{-func.)}
\end{align*}\]

The derivation in (45) proceeds as follows:

1. The tough-predicate fun combines with the CP and the degree variable to form an AP denoting a proposition.
2. \(\text{App}^0\) takes the AP as its propositional argument, returning a property of individuals wherein the judge of the proposition denoted by AP is shifted to the unsaturated individual argument \(\lambda j''\) of \(\text{App}^0\).
3. This individual argument is saturated with the experiencer PP.
4. Last, existential closure applies over the degree variable.

\[\Rightarrow\text{ Crucially, the AP in (45) denotes a proposition and is of the correct semantic type to compose with } \text{App}^0. \text{ As a result, the expletive construction can occur with an experiencer PP between the adjective and the embedded clause.}\]

✗ Tough-construction
However, in the tough-construction, \(\text{App}^0\) is unable to combine with the AP because it denotes a property of individuals. This creates an irreversable semantic-type mismatch.

\[
(46) \quad * \text{Hockey is fun } [\text{for Lucy}] [\text{Op}, \text{PRO to play } t_i ]
\]

\[
\exists aP_3 \quad \exists aP_2 \\
\text{hockey} \quad \text{aP}_1 \\
\text{App}^0 \quad \text{App}^0 \quad \text{X} \leftarrow \text{Type mismatch!}
\]

\[
\begin{align*}
\text{aP}_3 : st \\
\text{PP} \quad \text{App}^0 : (e, st) \quad \text{AP}_2 : (e, st) \\
\text{fun}_{\text{rc}} \quad \text{CP} : (e, st) \quad \text{PRO to play } \_ \_ \\
\text{Op, PRO to play } \_ \_
\end{align*}
\]
• The derivation in (46) proceeds as follows:
  1. The tough-predicate fun combines with the CP and the degree variable to form an AP denoting a property of individuals.
  2. Next, Appl\(^0\) tries to combine with the AP. Appl\(^0\) wants a propositional argument, but the AP denotes a property. With no way to semantically compose these two elements, the derivation crashes.

• Other modes of composition?—No.
  There are two nonstandard modes of composition that would in principle allow Appl\(^0\) and AP to compose semantically:
  1. Geach Rule (Geach 1972)
     This type-shifting rule would raise \(\langle st, \langle e, st \rangle \rangle\) to \(\langle \langle e, st \rangle, \langle e, \langle e, st \rangle \rangle \rangle\), allowing Appl\(^0\) to take AP as its argument via Function Application. However, this is in effect a decomposed version of Function Composition (see Jacobson 1999). We make the common assumption that Function Composition is not (freely) available in the syntax, decomposed or not. Therefore, a type-shifting rule like the Geach Rule is unavailable to repair the type mismatch.
  2. Restrict and flexible reordering of \(\lambda\)-abstractions (Chung & Ladusaw 2004)
     This would allow the AP to restrict, but not saturate the individual argument of Appl\(^0\). However, this would yield the wrong meaning, e.g. in (46) that it is fun to play Lucy. Moreover, it would still result in a type mismatch upon encountering the tough-subject, e.g. hockey in (46).

• Consequences
  This analysis accounts for the other properties of PP intervention as well:

  X Adjuncts also crash the semantics
  The same kind of semantic-type mismatch in (46) occurs for intervening adjuncts as well because they are of type \(\langle st, st \rangle\) (ignoring tense) and want a propositional argument. The AP in a tough-construction, which denotes a property of individuals, therefore cannot compose with these adjuncts:

  \[
  \begin{array}{c}
  \text{Adjunct} \\
  \langle st, st \rangle \\
  \end{array} \\
  \begin{array}{c}
  \text{AP} \\
  \langle e, st \rangle \\
  \vdash \\
  \text{fun} \ [ \text{Op}, \text{to play} \_ \_ \_] \\
  \end{array}
  \]

  X Interveners can occur elsewhere
  Although experiencer PPs and adjuncts cannot occur between the adjective and the embedded clause, our analysis predicts that they should be able to attach in the structure to a higher node if that node denotes a proposition. This prediction bears out:

  (48) (To Mary) cholesterol is important (*to Mary) to avoid (to Mary)
  (49) (At XMas) cholesterol is important (*at XMas) to avoid (at XMas)

  In (48) and (49), what would intervene between the adjective and the embedded clause can in fact attach at the edge of the matrix clause—because this node denotes a proposition.

  ✓ Argument PPs do not intervene
  As mentioned earlier, argument PPs do not intervene in a GDP when they occur between the adjective and the embedded clause. This is because they compose with the adjective before the embedded clause does:

  (50) John is too [ fond [ of Mary ] ] [ Op; to like \_ \_ \_ ]

  – By composing with the adjective before it composes with the embedded clause, argument PPs thereby avoid intervening in the semantic-composition process.

4.4 Section summary

• Tough-predicates come in two variants: (i) a proposition-taking version, which corresponds to the expletive construction, and a (ii) property-taking version, which corresponds to the tough-construction.

• In a tough-construction, the embedded clause is a null-operator structure wherein a null operator \(\overline{X}\)-moves from the gap position to the clause edge triggering abstraction over an individual variable.

• Experiencer PPs and adjuncts both yield an irresolvable semantic-type mismatch in a tough-construction because the AP denotes a property, not a proposition. This prevents the intervener from occurring between the embedded clause and the adjective.

⇒ Upshot
  This analysis accounts for the various intricacies uncovered in section 3 above:
  1. PPs do not cause intervention in structures that unambiguously involve A-movement in English.

  ⇒ English A-movement is not subject to PP intervention
2. PP intervention also arises in structures that lack an expletive counterpart and are thus not derived by A-movement.
   \[ \rightarrow \text{PP intervention indirectly results from the presence of a null operator in these constructions} \]

3. Adjunct PPs that demonstrably do not intervene for A-movement nonetheless give rise to intervention in tough-constructions.
   \[ \rightarrow \text{Adjuncts are not A-interveners but semantically require propositions} \]

4. PPs do not intervene if they are an argument of the tough-predicate.
   \[ \rightarrow \text{Argument PPs are subcategorized for by the adjective and can hence be semantically integrated} \]

5 Extensions and further issues

- The base-generation analysis that we propose extends beyond the intervention facts we used to motivate it. In particular, it handles without further ado a number of properties of tough-constructions which have in part proven elusive to long-movement accounts.

5.1 No reconstruction

- It is well-known that the tough-subject cannot take scope under the tough-predicate (Postal 1974), as in (51a). Since A-movement can at least in principle reconstruct (51b), this should be a possibility according to long-movement accounts.

(51) a. Only wide scope in tough-constructions:
   Someone is difficult to please.  \((\text{someone} \gg \text{difficult}; \ast \text{difficult} \gg \text{someone})\)

b. Low scope possible in A-raising:
   Someone seems to be sick.  \((\text{someone} \gg \text{seems}; \text{seems} \gg \text{someone})\)

- This is unproblematic on a base-generation account: There is no reconstruction because there is no movement.

5.2 No improper movement

- Long-movement accounts standardly postulate an A-\(\overline{A}\)-A movement chain (e.g. Hartman 2011), a configuration known as hyperraising, an instance of improper movement, which is otherwise ungrammatical:

(52) *John seems [CP t [TP t likes Mary ]]

- Problem for long-movement accounts
   Why the same movement chain should be well-formed in tough-constructions, but ungrammatical everywhere else is a long-standing problem for long-movement analyses.

- A smuggling derivation?
   In a recent attempt to resolve this paradox, Hicks (2009) postulates a smuggling account: The overt DP and the null operator form a complex DP in the base position. This complex DP raises to the embedded [Spec, CP]. The overt DP is then subextrated to the matrix [Spec, TP] position.

(53) \[ \text{John}_1 \text{ is tough } [\text{Op} t_1]_2 \text{ to please } t_2 \]

- Problem for smuggling account
   Though this analysis does not involve an A-\(\overline{A}\)-A chain, this revised derivation is likewise ungrammatical outside of tough-constructions (Abels 2007):

(54) *Oscar \(_1\) was asked [[how likely \(_1\) to win \(_3\) it was \(_2\)]

- Conclusion
   There does not currently exist a long-movement theory of tough-constructions that reconciles it independently observable constraints on A-movement. This is unproblematic on a base-generation account as it does not invoke any A-movement.
5.3 No crossover

- *Tough*-constructions exhibit a long-noticed by rarely addressed paradox: Despite the compelling evidence for \( \bar{A} \)-movement within the infinitival clause, this movement does not trigger weak crossover effects (Lasnik & Stowell 1991).

(55) Every man, should be easy for his wife to love.

- On a long-movement account, (56) has to involve \( \bar{A} \)-movement of *every man* over *his*. This crossover should make the example ungrammatical.

(56) Every man(i) should be easy [ \( t_i \) for \( \bar{A} \) his(i) wife to love \( t_i \) ]

- Under a base-generation account, this lack of weak crossover effects is unsurprising. As null-operator structures are interpreted via Predicate Abstraction, all that is necessary is that the tough-subject be coindexed with the bound pronoun and the operator be coindexed with its trace:

(57) Every man(i) should be easy [ \( \text{OP}_j \) for \( \bar{A} \) his(i) wife to love \( t_j \) ]

- Crucially, the operator does not need to be coindexed with its associate to achieve the correct interpretation. There is hence no crossover in the lower clause in (57) and (55) is correctly predicted to be grammatical.

⇒ Long movement accounts

Special assumptions about weak crossover and/or the relation between the movement steps are required on a long movement account.

5.4 Semantic differences

- Alleged synonymy

The main intuition underlying a movement approach to *tough*-constructions is that they are synonymous to the expletive construction. Yet there are cases where this synonymy breaks down. While *tough*-constructions can appear in the progressive, their expletive counterpart cannot (Lasnik & Fiengo 1974):

(58) Progressive in *tough*-constructions only

a. John is being easy to please ___.

b. *It is being easy to please John.

- Because our account rests on a semantic difference between the two constructions, it is well-equipped to handle this contrast.

- Semantics of the progressive

Partee (1977) suggests that progressive *be* combines with adjectives that can take an animate subject. It then ascribes a certain behavior to this subject:

(59) Informal denotation for progressive *be*

\[ \lambda \text{P}_{(e, st)} \lambda x \lambda w . x \text{ behaves in a way that } \text{P}(x)(w) \]

- It follows that the progressive head must apply to individuals and thus necessarily be of type \( \langle e, st \rangle, \langle e, st \rangle \). Consequently, it is incompatible with the expletive construction, which merely denotes a proposition (type st).

⇒ Conclusion: No synonymy

This account crucially requires the *tough*-frame and the expletive frame to be semantically distinct. It is hence out of reach on a long-movement analysis.

5.5 Predicate sensitivity

- An apparent disadvantage

The base-generation account requires two lexical entries for adjectives like *tough*. The long-movement analysis, on the other hand, seems to have the advantage of deriving one frame from the other.

- More classes of adjectives

Crucially, there are adjectives that can occur in the expletives but not in the *tough*-construction (e.g., *possible, polite*). As (60) shows, what frames an adjective allows is idiosyncratic. It is hence impossible to generally derive one frame from another.
(60) Adjective classes

<table>
<thead>
<tr>
<th></th>
<th>expletive construction</th>
<th>YES</th>
<th>NO</th>
</tr>
</thead>
<tbody>
<tr>
<td>tough-construction</td>
<td>tough, impossible, ...</td>
<td>pretty, tasty, ...</td>
<td>possible, polite, ...</td>
</tr>
</tbody>
</table>

* Conclusion

That tough can occur in both has to be explicitly stated under any analysis. A movement account is no more elegant than the base-generation analysis.

5.6 Agree-based accounts

- **Agree**
  
  Our base generation account links the matrix subject to the embedded gap by semantic means. Rezac (2006) suggests a syntactic version of the base generation account: Simplifying somewhat, the matrix T agrees with the embedded operator (also see Fleisher to appear).

- Rezac also argues that this process of Agree is active in copy raising constructions like (61):

  (61) John seems like he's the smartest guy in the world.

- Hartman (2011)'s intervention facts constitute an argument against an Agree analysis: Copy raising constructions are not subject to PP intervention:

  (62) John seems [PP to Mary ] like he's the smartest guy in the word.

- To the extent that copy raising allows us to independently diagnose the properties of the alleged Agree step, the PP intervention facts in tough-constructions are unaccounted for on this analysis.

5.7 **Tough-constructions beyond adjectives**

- Our account is phrased in terms of semantic properties, not syntactic ones. This leads one to expect parallel facts for predicates other than adjectives. This expectation is borne out.

- **Nouns**

  Nouns like pleasure behave identical to tough-predicates:

  (63) **Tough-like nouns**

      a. It is a pleasure to visit Reykjavik.

      b. Reykjavik is a pleasure to visit ____.

- Just like adjectives, some nouns occur in only one of the two constructions (Lasnik & Fiengo 1974):

  (64) **Nouns occurring in tough-constructions only**

      a. *It is a marvel to look at Kyle.

      b. Kyle is a marvel to look at ____.

  (65) **Nouns occurring in expletive construction only**

      a. It was a mistake to fire Bill.

      b. *Bill was a mistake to fire ____.

- **PP intervention**

  PP intervention effects also arise for these nominals in the tough-construction:

  (66) a. Mary is a beauty [PP to John ]

      b. Mary is a beauty ("[PP to John ]") to look at ____.
(67) Scenario:
Bill is terrible at math and at risk of failing his math class. His teacher, however, is extremely supportive and goes out of his way to help Bill pass the class. He comes up with all sorts of different teaching techniques to make Bill understand the materials. But nonetheless, it is exceedingly hard for the teacher to make Bill pass the class.

a. ?It is a challenge [PP for the teacher ] [PP for Bill ] to pass the class.
b. *This class, is a challenge [PP for the teacher ] [PP for Bill ] to pass ___i.

* Upshot
Our account extends to tough-like nouns and the corresponding intervention facts without further ado. A defective intervention would have to stipulate that PPs, but not DPs, act as defective interveners, a curious conspiracy.

5.8 Tough-predicates inside DPs

- It is often noted that tough+DP constructions are similar to tough-constructions and should be given a uniform account. The base generation account can be straightforwardly extended to accomplish this.

(68) John is a tough lawyer to beat ___

\[
 S \\
 John \quad DP \quad lawyer \quad AP \quad tough \ [ Op_i \ PRO_{\text{Ann}} \ \text{to beat} \ ___i \]
\]

a. \[ \text{[AP]}^j = \lambda x \lambda w . \exists d \forall \{w', j\} \in \text{ACC}_{w,j} \ \text{[tough}(x)(j')(w')\text )(d) \]

b. \[ \text{[DP]}^j = \lambda x \lambda w . \text{LAWYER}(x)(w) \land \exists d \forall \{w', j\} \in \text{ACC}_{w,j} \ \text{[tough}(x)(j')(w')\text )(d) \]

- In (68), as lawyer and tough to beat are both of type (e, st), they can combine via simple Predicate Modification.

6 Conclusion

- Hartman (2011, 2012a,b) provides evidence that in tough-constructions the tough-predicate cannot take an experiencer PP and argues that this provides evidence for a long-movement account of tough-constructions.

- We have argued that this restriction is part of a larger generalisation: In null operator structures (tough-constructions, pretty-predicate, GDP), no adjunct can appear between the adjective and the infinitival clause. This larger pattern remains largely unaccounted for on a long-movement account.

- We proposed a base-generation account of tough-constructions that allows for a more comprehensive account of the intervention facts: In tough-constructions, an intervening adjuncts creates an unresolvable type mismatch.

- This account derives, without further machinery, several other well-known properties of tough-constructions.

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Heim, Irene. 1982. The semantics of definite and indefinite noun phrases. Doctoral Dissertation, University of Massachusetts Amherst, Amherst, MA.
Appendix A: Predicates of personal taste and control

- **Semantics of predicates of personal taste**
  Stephenson (2007, 2010) proposes that predicates of personal taste are two-place predicates, following Lasersohn (2005):

  \[(\text{tasty}) = \lambda y \cdot \lambda x \cdot \lambda w \cdot . \ x \ \text{tastes good to } y \ \text{in } w\]

- The first argument \(\lambda y\) corresponds to the judge. It can be saturated in one of two ways: (i) A silent referential pronoun \(\text{pro}\) \((70)\) and (ii) \(\text{PRO}\) \((71)\). Stephenson argues that \(\text{PRO}\) refers directly to the judge of the proposition, represented as the argument \(j\) of the denotation function:

  \[(\text{tasty pro}_\text{John}) = [\lambda y \cdot \lambda x \cdot \lambda w \cdot . \ x \ \text{tastes good to } y \ \text{in } w] \ (\text{John}) = \lambda x \cdot \lambda w \cdot . \ x \ \text{tastes good to } \text{John in } w\]

  \[(\text{tasty PRO}) = [\lambda y \cdot \lambda x \cdot \lambda w \cdot . \ x \ \text{tastes good to } y \ \text{in } w] \ (\text{PRO}) = [\lambda y \cdot \lambda x \cdot \lambda w \cdot . \ x \ \text{tastes good to } y \ \text{in } w] \ (j) = \lambda x \cdot \lambda w \cdot . \ x \ \text{tastes good to } j \ \text{in } w\]

- Having \(\text{PRO}\) refer directly to the judge allows us to capture the fact that the judge of a predicate of personal taste can refer to an attitude holder when it is embedded under an attitude predicate:

  \[(\text{want}) = \lambda p \cdot \lambda x \cdot \lambda w \cdot . \ \forall (w', y) \in \text{want}_{w,x} [\lambda p] \ (w')\]

  \[(\text{want}_{x,y}) = ((w', y) : \text{it is compatible with what } x \ \text{wants in } w \ \text{for } x \ \text{to be } y \ \text{in } w')\]

  Sue wants [the cake to be [tasty \(\text{PRO}\)]]

  \(\text{a. } [\text{want}]' ((\text{the cake to be [tasty \(\text{PRO}\)]}) \ (Sue))\)

  \(\text{b. } 1 \text{ iff } \forall (w', y) \in \text{want}_{w,Sue} \ [\text{the cake tastes good to } y \ \text{in } w']\)

- **Semantics of control**
  Stephenson extends her proposal that \(\text{PRO}\) refers to the judge to account for the obligatory *de se* interpretation of \(\text{PRO}\) under attitude predicates:

  \[(\text{want}) = \lambda p \cdot \lambda x \cdot \lambda w \cdot . \ \forall (w', y) \in \text{want}_{w,x} [\lambda p] \ (w')\]

  \[(\text{want}_{x,y}) = ((w', y) : \text{it is compatible with what } x \ \text{wants in } w \ \text{for } x \ \text{to be } y \ \text{in } w')\]

  Sue wants [\(\text{PRO}\) to go on the roller coaster]

  \(\text{a. } [\text{want}]' ((\text{PRO to go on the roller coaster}) \ (Sue))\)

  \(\text{b. } 1 \text{ iff } \forall (w', y) \in \text{want}_{w,Sue} [y \ \text{goes on the roller coaster in } w']\)

Appendix B: Gapped Degree Phrases

- **Frozen scope → In-situ**
  Heim (2001) observes that the *too*-clause can scope over an intensional verb only in the gapless variant:

  \[(\text{too}) = \lambda x \cdot \lambda w \cdot . \ \exists d \ [f (d)(w') \ ∧ \exists w' \in \text{Acc}_{w} [f (d)(w') ∧ \text{LIFT} (x) (w')]]\]

- **Faraci’s generalisation → In-situ**
  Faraci (1974) observes that the gap must be coreferential with subject argument of the adjective:

  \[(\text{too}) = \lambda x \cdot \lambda w \cdot . \ \exists d \ [f (d)(w') \ ∧ \exists w' \in \text{Acc}_{w} [f (d)(w') ∧ \text{LIFT} (x) (w')]]\]

- **Semantics of GDVs**
  Nissenbaum & Schwarz (2011) analyse GDVs as null-operator structures. These structures are interpreted via compose, a semantic operation that exhaustively applies function application (FA) and predicate modification (PM) to its two arguments (Nissenbaum 2000). When applied to a GDP, compose yields an \((e, st)-function\) which then applies to the base-generated matrix subject:

  \[(\text{too}) = \lambda x \cdot \lambda w \cdot . \ \exists d \ [f (d)(w') \ ∧ \exists w' \in \text{Acc}_{w} [f (d)(w') ∧ \text{LIFT} (x) (w')]]\]

  \(\text{a. } \text{The table is too heavy} [\text{OP}_1 \ [\text{to lift } t]] \ \sim \text{LF: The table is [heavy] [\lambda x [\text{too to lift } x]]}]\)

  \(\text{b. } [\text{heavy}] = \lambda x \cdot \lambda d \cdot \lambda w \cdot \cdot \cdot \geq d \ [\text{OP}_1 \ [\text{too to lift } t]] = \lambda x \cdot \lambda f (d) [f/d] \cdot \lambda w \cdot \not\exists d \ [f (d)(w') \ ∧ \not\exists w' \in \text{Acc}_{w} [f (d)(w') \ ∧ \text{LIFT} (x) (w')]]\)

  \(\text{c. } \text{compose} = \lambda x \cdot \lambda w \cdot [\lambda x [\text{too to lift } x]] (\text{heavy}) = \text{compose} \ (\text{e, st}) \text{ (by PM)}\)

  \(\text{d. } \lambda y \cdot \text{compose} = \lambda y \cdot [\lambda x [\text{too to lift } x]] (y) (\text{heavy} (y)) = \text{compose} \ (\text{e, st}) \text{ (by FA)}\)

  \(\lambda y \cdot [\lambda x [\text{too to lift } x]] (y) (\text{heavy} (y))\)